Revenue Enhancement Via Asymmetric Signaling in Interdependent-Value Auctions

Zhuoshu Li and **Sanmay Das***
Washington University in St. Louis
{zhuoshuli, sanmay}@wustl.edu

Abstract

We consider the problem of designing the information environment for revenue maximization in a sealed-bid second price auction with two bidders. Much of the prior literature has focused on signal design in settings where bidders are symmetrically informed, or on the design of optimal mechanisms under fixed information structures. We study common- and interdependent-value settings where the mechanism is fixed (a second-price auction), but the auctioneer controls the signal structure for bidders. We show that in a standard common-value auction setting, there is no benefit to the auctioneer in terms of expected revenue from sharing information with the bidders, although there are effects on the distribution of revenues. In an interdependent-value model with mixed private- and common-value components, however, we show that asymmetric, information-revealing signals can increase revenue.

Introduction

In most of the literature on mechanism design, the model assumes that agents' information is given, and searches for rules of the game that yield desired outcomes. However, there has recently been considerable interest in the parallel problem of designing the *information environment* that agents will encounter (Kamenica and Gentzkow 2011; Das and Kamenica 2015). This paradigm is clearly applicable in many scenarios of interest to AI researchers, including online advertising, internet marketplaces, and so on. One particular domain where this is interesting is in auctions with signaling, which have been studied extensively in both economics and computer science. Assume a fixed mechanism; can the seller expect to make more revenue if the bidders are more or less informed than the "baseline"?

Auctions with signaling have been studied in several different contexts. Much of the literature assumes that agents are symmetric with respect to the information they receive about the value of the item, in the sense that the bidders' signals are drawn from the same distribution. For example, the seminal "Linkage Principal" of Milgrom and Weber (1982a) states that fully and publicly announcing all information available to the seller is the expected-revenue-maximizing policy in common value auctions. Somewhat less is known about auctions with asymmetrically informed bidders, and most of that literature has focused on understanding how information asymmetries affect revenue rather than on the design of the optimal signal structure. There has also been a line of work on so-called "deliberative auctions" (Larson and Sandholm 2005; Brinkman, Wellman, and Page 2014), where agents have the opportunity to acquire information about valuations before entering a bidding process. Most of this literature focuses on strategic choices by the bidders and how this affects equilibrium outcomes of the auction.

Here we analyze signal design in auctions as a persuasion game, following Kamenica and Gentzkow (2011) who consider the problem of designing the optimal information environment for the case between one self-interested agent ("sender") and one decision-maker ("receiver"), where both of them are rational Bayesians. The sender can design the *information structure* or *signal structure* to release information about the state of the world to receiver before the receiver makes her choice. In the auction setting, the signal structure induces a game between the bidders, and the equilibrium outcome of the game affects the seller's revenue.

Our contribution We consider a sealed-bid second price auction with two bidders. As usual, the winner is the bidder who submitted the highest bid (with ties broken equiprobably in either direction), but pays to the seller the second highest bid. The bidders and seller share the same common prior on the underlying state of the item. Before the bidding stage, the seller can provide a (noisy) signal to each bidder based on the state of the world. She commits to a signaling strategy in advance, and the resultant structure becomes common knowledge. We explore the following two auction games: (1) a basic common-value auction model, where the value of the item is determined either by a single attribute or by two independent attributes when each bidder can receive information from exactly one of the attributes; (2) an interdependent-value auction, where the valuation for each bidder is decided by a common value attribute and a private attribute. We show that in the common-value auction settings, there is no benefit to the auctioneer in terms of expected revenue from sharing information with the bidders, although there are effects on the distribution of revenues. In an interdependent-value model with mixed private- and

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common-value components, however, we show that asymmetric, information-revealing signals can increase revenue.

Our model contributes to the growing literature on Bayesian persuasion with multiple receivers; this literature usually focuses on public signals (Dughmi 2018, e.g.) or symmetric signal structures (Das, Kamenica, and Mirka 2017, e.g.). Our model is applicable to complex situations where the sender of the signal has the opportunity to communicate in different ways to different receivers. This can happen in situations like corporate mergers (Berkovitch and Khanna 1991; Reuer, Tong, and Wu 2012), where targets (sellers or signal senders in our case) have to communicate with potential acquirers (the signal receivers). It is known that targets often inflate their output (Gilson and Schwartz 2005) or themselves may not be aware of their value to an acquirer due to the complexity and intangible characteristics which cannot be easily observed (Knecht and Calenbuhr 2007).

Related work

Our work is related to several literatures. Broadly, this paper fits into a growing line of literature in AI on how the information environment available to agents influences market outcomes. Hajaj and Sarne (2014) examine how e-commerce platforms can gain from information withholding policies. Chhabra *et al* (2014) study the welfare effects of competition between information providers with different levels of information quality. Das and Li (2014) model the effects of common and private signals about quality in matching with interviews. Rabinovich et al (2015) present an efficient model for security asset assignment which combines both Stackelberg security games and the Bayesian Persuasion model.

The literature on auctions with signaling, as mentioned above, typically analyzes symmetric information structures, where there are few positive results in terms of revenue enhancements. In addition to the literature from economics cited above, recent work in algorithmic economics that assumes symmetric information disclosure includes that of Emek et al (2012) as well as Bro Miltersen and Sheffet (2012), both of which study second-price auctions of multiple indivisible goods and consider hiding information by clustering. Guo and Deligkas (2013) presents single-item second-price auctions where the item is characterized by a set of attributes and the auctioneer decides whether to hide a subset of attributes.

When we move to asymmetric information, most early work considers the case in which one bidder is perfectly informed about the value of the item, while the other bidders are entirely uninformed (Wilson 1967; Milgrom and Weber 1982b). Milgrom and Weber (1982b) show that reducing information asymmetries can increase the seller's expected revenue in a two-bidder first-price common value auction where one bidder is perfectly informed and the other bidder is entirely uninformed. Goeree and Offerman (2003) also consider public information disclosure in common value auctions, in which the common value is an average of i.i.d. private values (signals) of all bidders. They also conclude that seller's public information disclosure can raises seller's revenues. Hausch (1987), however, through a simple example in a first price common value auction, shows that reducing information asymmetry may decrease the seller's expected revenue when the better-informed bidder is neither strictly better-informed nor perfectly informed.

Syrgkanis *et al* (2015) consider common value hybrid auctions where the payment is a weighted average of the highest and second-highest bids. They show that public revelation of an additional signal to both bidders may decrease the auctioneer's revenue, different from (Milgrom and Weber 1982a). Parreiras (2006) consider continuous signal spaces and also show that second price auction revenue-dominates first price auction. Bergemann *et al* (2017) provide revenue guarantees for the first price auction under different information structures. In all of these papers, the seller does not control the information structure for both bidders.

There are also several recent papers considering this question from the optimal mechanism design perspective (Skreta 2011; Bergemann and Pesendorfer 2007; Daskalakis, Papadimitriou, and Tzamos 2016), rather than assuming a fixed structure for the mechanism and analyzing the question of optimal signaling given the mechanism. Recent work of Alkoby et al (2017) analyzes signaling by a third party information provider under a fixed mechanism.

We position this work in the persuasion literature (Rayo and Segal 2010; Kamenica and Gentzkow 2011), where a sender strategically reveals information through signals. Much of this literature focuses on the design of the optimal signaling scheme (Ostrovsky and Schwarz 2010; Gentzkow and Kamenica 2016). While this is tractable in some cases, for example with costly signals and a single receiver (Gentzkow and Kamenica 2014), or when a single buyer is signaling to a single monopolist seller (Shen, Tang, and Zeng 2018), the problem of optimal signal design is not always even computationally, leave alone analytically, tractable (Dughmi 2018; Xu et al. 2016). Therefore, the demonstration of a revenue-enhancing signal structure in the game with multiple receivers that we demonstrate here is significant, even if the particular structure we find is not the optimal one.

Also related is the literature on deliberative auctions. Deliberation covers any actions that update an agent's belief. In the study of deliberative auctions, research has thus far focused on either the perspective of bidders (receivers) or on optimal mechanism design. Larson and Sandholm (2001a; 2001b) provide a very general model for costly information gathering in auctions. They show that under costly deliberation, bidders perform strategic deliberation in equilibrium in most standard auction settings (Vickrey, English, Dutch, first price and VCG). Thompson and Leyton-Brown (2007) investigate deliberation strategies for second price auctions where agents have independent private values (IPV) and the impact of agents' strategies on seller's revenue. They perform equilibrium analysis for (1) deliberation with costs, (2) free, but time-limited deliberation. They further show that, in the IPV deliberative-agent setting, the only dominant-strategy mechanism is a sequential posted price auction, in which bidders are sequentially given a posted-price, take-it-or-leave-it offer until the good is sold (Thompson and Leyton-Brown 2011). Celis *et al* (2012) provide an efficient mechanism in IPV deliberative-agent setting to obtain revenue within a small constant factor of the maximum possible revenue. Brinkman *et al* (2014) show that the dependence structures among agents' signals of the value of the item they are bidding on can produce qualitatively different equilibrium outcomes of the auction. This literature also typically does not focus on the optimal design of the signal structure from the perspective of the seller.

Common Value Auctions

We begin by considering a single-item auction with two risk-neutral bidders (agents) $i \in \{1, 2\}$ and a seller. Both bidders value the object identically: the item has a common value of $v \in R^+$ to the two bidders. The realization of v is not observed by either the seller or the bidders. v depends on an underlying state of the world $w \in \Omega$. Without loss of generality, we assume that the item's value is 0 when w's quality is *Bad (B)* and 1 when w's quality is *Good (G)*, and the common prior is represented by $P(G) = x, x \in [0, 1]$. Before bidding, each bidder receives a conditionally independent low (L), or high (H) signal from seller without cost, $s_i \in \{H, L\}$.

$$\begin{aligned} P[s_1 = H|G] = p_1 \quad P[s_1 = L|B] = q_1 \\ P[s_2 = H|G] = p_2 \quad P[s_2 = L|B] = q_2 \end{aligned}$$

where s_i is agent i's signal and all signals have accuracy of $p_i, q_i \in [1/2, 1]$. Thus, a high (low) signal suggests a good (bad) value of the item.

Following prior literature, we make some assumptions.

Assumption 1 *Seller cannot distort or conceal information once the signal realization is known (Kamenica and Gentzkow 2011).*

Assumption 2 *Bidders play only weakly undominated strategies (Brinkman, Wellman, and Page 2014).*

The first assumption allows us to abstract from the incentive compatibility issues, while the second helps rule out implausible or uninteresting equilibria.

In the game, the seller decides the signal structure S with the goal of maximizing her expected revenue R and the bidders submit their bids based on their private signals s_i. The seller runs a two-player second-price sealed-bid (SPSB) auction. Define $\text{bid}_{s_{-i}}(s_i)$ as the bid of bidder i given she receives signal s_i and the other bidder receives signal s_{-i}. The seller can either reveal the realization of the signal privately to the corresponding bidder, or reveal it publicly. Here we show the analysis of private revelation, as public revelation follows similarly.

Proposition 1 *If the seller reveals the realization of the signal privately to the corresponding bidder, a unique symmetric equilibrium exists. Each agent bids her expected value conditioned on her opponent's signal being equal to her own,*

$$\begin{aligned} bid_L(L) &= \mathbb{E}[v|s_1 = L, s_2 = L] \\ &= P(G|s_1 = L, s_2 = L) \\ &= \frac{(1-p_1)(1-p_2)x}{(1-p_1)(1-p_2)x + q_1q_2(1-x)}, \end{aligned}$$

$$\begin{aligned} bid_H(H) &= \mathbb{E}[v|s_1 = H, s_2 = H] \\ &= P(G|s_1 = H, s_2 = H) \\ &= \frac{p_1p_2x}{p_1p_2x + (1-q_1)(1-q_2)(1-x)}. \end{aligned}$$

Proof The proof of this proposition is similar to prior work of Hausch (1987) and of Brinkman, Wellman, and Page (2014). Assumption 2 (that bidders play only weakly undominated strategies) restricts an agent with a *Low* signal to bid between $E[v|s_i = L, s_{-i} = L]$ and $E[v|s_i = L, s_{-i} = H]$, and one with a *High* signal to bid between $E[v|s_i = H, s_{-i} = L]$ and $E[v|s_i = H, s_{-i} = H]$. To see that the proposed strategy in proposition 1 is the only symmetric equilibrium, we begin by assuming that there exists a symmetric strategy that, when receiving signal L, the Bidder 1 bids x_1 and the Bidder 2 bids x_2, and when receiving signal H, Bidder 1 bids y_1 and Bidder 2 bids y_2. Suppose $x_1 \geq x_2$, then Bidder 1 will be strictly better off by deviating to $E[v|s_i = L, s_{-i} = L]$ when receiving an L signal, since bidding x_1 could result in negative utility ($E[v|s_i = L, s_{-i} = L] - x_2$) if Bidder 2 also receives an L signal. Similarly, if $y_1 \geq y_2$, Bidder 2 has incentive to switch to $E[v|s_i = H, s_{-i} = H]$ when receiving an H signal to achieve higher expected utility. Thus, the equilibrium bids above constitute the only symmetric equilibrium. □

Equilibrium selection It is well known that the second-price common-value auction generally has many equilibria (Krishna 2002; Hausch 1987; Abraham et al. 2011, and so on). Assumption 2 helps us to rule out all dominated bids. In this game, suppose that Bidder 1 obeys the strategy in Proposition 1. Bidder 2, conditional on receiving signal L bids $b \in (\text{bid}_L(L), \mathbb{E}[v|s_1 = H, s_2 = L]]$ and, conditional on receiving signal H, bids $\text{bid}_H(H)$. These strategies are still Nash equilibria. Thus, Nash equilibrium provides no prediction about revenue beyond an upper bound on the full surplus. For this paper's purpose, therefore, we only focus on symmetric equilibrium bidding strategies.

In a common value auction, the seller's expected revenue R is the expected value $\mathbb{E}[v]$ of the item, minus the sum of the two bidders' utilities. When each bidder observes a private signal only, we can treat each bidder independently and minimize the utility of each bidder.

Theorem 1 *If each bidder observes her own private signal, the optimal signal structure for the seller in terms of revenue is* $p_1 = p_2 = 1, q_1, q_2 \in [1/2, 1]$, *or* $p_1 = p_2 = q_1 = q_2 = 1/2$, *where* $\max R = \mathbb{E}[v]$.

Proof For revenue maximization, we can treat the two-bidder second-price sealed-bid auction as a three-player, constant-sum game. The revenue

$$R = \mathbb{E}[v] - \mathbb{E}[u_1] - \mathbb{E}[u_2]. \tag{1}$$

$\mathbb{E}[u_i]$ is Bidder i's expected utility and R is maximized when $\mathbb{E}[u_1] = \mathbb{E}[u_2] = 0$, where

$$\begin{aligned} \mathbb{E}[u_1] &= p(s_1 = H, s_2 = L)(\mathbb{E}[v|s_1 = H, s_2 = L] - \text{bid}_L(L)) \\ &= p_1(1-p_2)x - p(s_1 = H, s_2 = L)\text{bid}_L(L), \\ \mathbb{E}[u_2] &= p(s_1 = L, s_2 = H)(\mathbb{E}[v|s_1 = L, s_2 = H] - \text{bid}_L(L)) \\ &= (1-p_1)p_2x - p(s_1 = L, s_2 = H)\text{bid}_L(L), \end{aligned}$$

which gives us $p_1 = p_2 = 1, q_1, q_2 \in [\frac{1}{2}, 1], \forall x \in [0,1]$, or $p_1 = p_2 = q_1 = q_2 = \frac{1}{2}, \forall x \in [0,1]$, or $p_1, p_2, q_1, q_2 \in [\frac{1}{2}, 1]$, when $x = 1$. When $p_1 = p_2 = q_1 = q_2 = 1$, the seller always reveals complete information, thus the expected revenue R is also $\mathbb{E}[v]$. □

Another natural question to ask concerns the distribution of revenues to the seller under different signal structures. It is relatively easy to compute the variance of the revenue

$$var(R) = (\text{bid}_L(L) - \text{bid}_H(H))^2 (1 - P(s_1 = H, s_2 = H)) \\ P(s_1 = H, s_2 = H) \tag{2}$$

Clearly $var(R)$ is minimized at $q_1 = q_2 = 0.5$. Figure 1 shows some illustrative examples of the standard deviation of revenue.

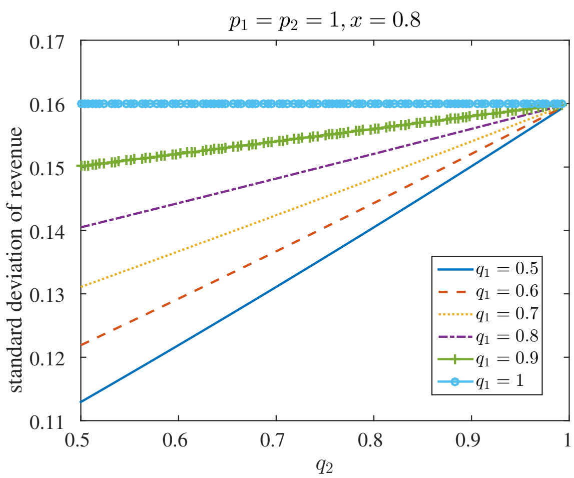

Figure 1: Standard deviations of revenue for different revenue-maximizing signal structures in the simple common-value model. While each of these signal structures achieves the same revenue, the risk profiles are substantially different.

Adding an Intermediate Value

Brinkman *et al* (2014) study a common-value auction setting with intermediate values, which serves as a model for studying signal acquisition by bidders. They motivate this setting with an example of the auction of extraction rights for some resources (say oil and gas) on a specified plot of land. The value to energy companies of these rights depends on the unknown amounts of extractable resources. The question of optimal signaling is motivated in this example by the fact that the government can reveal information about one or both of the specific resources to each energy company. Now the item can take on three possible values, $\{0, g, 1\}$ with $g \in [0,1]$. The underlying state w which decides the value of the item now has two attributes, $w = (w_1, w_2)$. Each attribute is associated with signals potentially observed by the respective agents. Each bidder can request one signal with no cost. Here we study a variant where the seller can decide which attribute to signal to each bidder and what the corresponding signal structure should be.

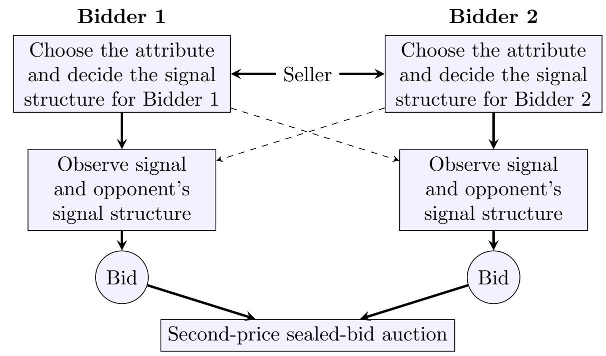

Figure 2: The intermediate-value model. Dashed lines mean that the bidder knows the structure of the signal that the other bidder receives, but not the specific realization.

Each attribute is still either *Good (G)* or *Bad (B)*, where $P(w_j = G) = x \in [0,1], j \in \{1,2\}$. The realization of each signal is also *High (H)* or *Low (L)*. The signal structure can be represented as ($s_i^j \in \{H, L\}$):

$$P[s_1^j = H | w_j = G] = p_1 \quad P[s_1^j = L | w_j = B] = q_1$$
$$P[s_2^j = H | w_j = G] = p_2 \quad P[s_2^j = L | w_j = B] = q_2$$

where $j \in \{1,2\}$ and s_i^j is Bidder i's signal from attribute j. All signals have accuracy of $p_i, q_i \in [1/2, 1]$.

The value of the good is 0 if neither attribute is G, 1 if both are G, and $g \in [0,1]$ if only one is G.

$$v = \begin{cases} 0, \text{if } \sum_j \mathbb{I}\{w_j = G\} = 0 \\ g, \text{if } \sum_j \mathbb{I}\{w_j = G\} = 1 \\ 1, \text{if } \sum_j \mathbb{I}\{w_j = G\} = 2 \end{cases}$$

Figure 2 shows the decision flow in this game. The seller's goal is to maximize her expected revenue R. The signal structure and the seller's choice of which attribute to signal to each bidder are both common knowledge.

First, we observe that it must again be the case that the seller's revenue is maximized when revealing no information even in this intermediate value setting, since it can still be modeled as a three-player, constant-sum game, and Equation (1) holds. What can we say about signal structures that achieve this revenue? Again, we analyze private revelation.

Theorem 2 *In the intermediate value model, (1) if the seller sends signals of different attributes to the two buyers, there is only one signal structure,* $\forall g, x \in [0,1]$, $p_1 = p_2 = q_1 = q_2 = 1/2$ *(equivalent to sending no information) that achieves the maximum possible revenue; (2) if the seller sends signals of the same attribute to both buyers, for* $\forall g, x \in [0,1]$, *there are a number of signal structures that achieve the maximum possible revenue:* $p_1 = p_2 = 1, q_1, q_2 \in [1/2, 1]$ *or* $p_1 = p_2 = q_1 = q_2 = 1/2$.

Proof The seller's revenue still follows Equation (1). To maximize R, $\mathbb{E}[u_1] = \mathbb{E}[u_2] = 0$.

- *Sending signals of the same attribute:*
  The unique symmetric equilibrium bidding strategy is that each bidder bids her expected value conditioned on her opponent's signal being equal to her own,
  $$\text{bid}_L(L) = \mathbb{E}(v|s_i^j = L, s_{-i}^j = L),$$
  $$\text{bid}_H(H) = \mathbb{E}(v|s_i^j = H, s_{-i}^j = H).$$
  We denote $P(s_i^j = H, s_{-i}^j = L)$ by $P(HL)$,
  $$\mathbb{E}[u_i] = P(HL)(\mathbb{E}(v|s_i^j = H, s_{-i}^j = L) - \text{bid}_L(L)).$$
  Thus, to maximize R
  $$\mathbb{E}(v|s_i^j = H, s_{-i}^j = L) = \text{bid}_L(L). \quad (3)$$
  The solution of Equation (3) is $p_1 = p_2 = q_1 = q_2 = \frac{1}{2}, \forall g, x \in [0,1]$, or $p_1 = p_2 = 1, q_1, q_2 \in [\frac{1}{2}, 1], \forall g, x \in [0,1]$, or $p_1, p_2, q_1, q_2 \in [\frac{1}{2}, 1]$, when $x = 1, \forall g \in [0,1]$. When $p_1 = p_2 = q_1 = q_2 = 1$, the seller reveals perfect information, thus the expected revenue R is also $\mathbb{E}[v]$.
- *Sending signals of different attributes:*
  As the signal accuracy between different attribute is identical, the equilibrium biding strategy is same as above, that is to bid the expected valuation conditioned on the opponent observing the same signal value. Denote $\text{bid}_{-s_{-i}}(s_i)$ as the bid of Bidder i given she receives s_i and the other bidder observes the signal of the other attribute and receives signal s_{-i} ,
  $$\text{bid}_{-L}(L) = \mathbb{E}(v|s_i^j = L, s_{-i}^{-j} = L),$$
  $$\text{bid}_{-H}(H) = \mathbb{E}(v|s_i^j = H, s_{-i}^{-j} = H).$$
  We simplify $P(s_i^j = H, s_{-i}^{-j} = L)$ by $P(H, L)$,
  $$\mathbb{E} = [u_i] = P(H,L)(\mathbb{E}(v|s_i^j = H, s_{-i}^{-j} = L) - \text{bid}_{-L}(L)).$$
  Thus, to maximize R,
  $$\mathbb{E}(v|s_i^j = H, s_{-i}^{-j} = L) = \text{bid}_{-L}(L). \quad (4)$$
  Solving Equation (4) we get,
  $$p_1 = p_2 = q_1 = q_2 = \frac{1}{2}, \forall g, x \in [0,1]. \quad \square$$

It is again easy to show that $var(R)$ is minimized at $q_1 = q_2 = 0.5$.

Discussion Brinkman *et al* (2014) analyze this problem from the perspective of the bidders. In their model, the signal structure is fixed and restricted to the symmetric information case ($p_1 = p_2 = q_1 = q_2$). They show that when the two attributes are sufficiently complementary, that is $g \to 0$, and the signals are noisy, the agents choose to observe the same attribute. When the signal accuracy is high, or the two signals are substitutable $g \to 1$, the agents choose to observe different attributes. Our result above demonstrates that, from the seller's perspective, sending no information can always maximize seller's expected revenue. The seller can also achieve the maximum possible revenue by sending information on the same attribute to both bidders. The corresponding signal structure shows that the bidders always know the item is bad if they see a *low* signal, but they have uncertainty when they see a *high* signal.

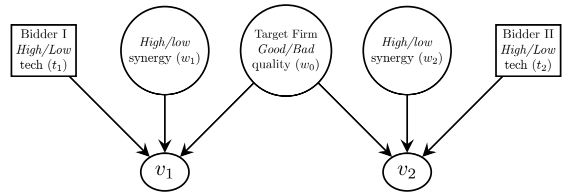

Figure 3: A sketch of the interdependent value setting.

An Interdependent Value Auction

We now move to a setting with an unambiguously positive result for the seller. We consider a classic situation in corporate mergers. A firm (target) can generate synergies if acquired by another firm (bidder) (Berkovitch and Khanna 1991). The source of this synergy may include management, economies of scale, technological matches, tax savings, etc. A sketch of the game is shown in Figure 3. The target's quality can be either good or bad, which is unknown to the market and the bidders at the time of bidding. The bidders' types can be high or low tech, privately known to each bidder. The ability of a bidder to generate synergies can be either high or low, which is unknown to the market and to the bidders, but may be discovered by the target (since the target is willing to invest in discovering this prior to making it known that it is open to acquisition). If the type of a bidder is high tech, as long as the ability of the bidder to generate synergies is high, it can get high value ($\alpha > 1$) no matter the target's quality. However, if the type of a bidder is low tech, only when both the ability of the bidder to generate synergies is high and the quality of the target is good, can it get medium value (1).

Model We first extend the common-value model of Brinkman *et al* to this situation. The item's value still depends on an underlying state w, which now has three attributes $w = (w_0, w_1, w_2)$. The common attribute w_0 can affect the valuation of both bidders (quality of the target firm), and the private attributes w_1 and w_2 only affect each bidder's own valuation respectively (idiosyncratic synergies). Each attribute takes quality *Good (G)* or *Bad (B)* as above. For simplicity, we assume $P(w_j = G) = x \in [0,1], j \in \{0,1,2\}$ (this assumption can be easily removed and all results hold). The seller sends a signal of the quality of either common or private attribute w_j to each bidder. The realization of each signal is also *High (H)* or *Low (L)*. The signal structure is ($s_i^j \in \{H, L\}$)

$$P[s_1^j = H|w_j = G] = p_1, \quad P[s_1^j = L|w_j = B] = q_1,$$
$$P[s_2^j = H|w_j = G] = p_2, \quad P[s_2^j = L|w_j = B] = q_2.$$

All signals have accuracy of $p_i, q_i \in [1/2, 1]$. Once the signal structure is decided, it becomes common knowledge. The seller can choose to either reveal realizations publicly or privately.

The bidders can be of two types, $t_i \in \{t_l, t_h\}$. The bidders will be of either type with probability $P(t_i = t_l) = P(t_i =$

Case	c_1	pr_1	c_2	pr_2	Revenue impact	Maximizing structure	Remarks
1	no	no	no	no	-	-	
2	no	yes	no	no	$\downarrow$	no information	unique eq
3	no	yes	no	yes	$\downarrow$	no information	unique eq
4	publicly	no	no	no	-	any	unique eq
5	publicly	no	publicly	no	-	any	unique eq
6	publicly	no	no	yes	$\downarrow$	private signal no information	unique eq
7	publicly	no	privately	no	$\downarrow$	lower bound maximized at no information	multiple eqs
8	privately	no	no	no	$\downarrow$	lower bound maximized at no information	multiple eqs
9	privately	no	no	yes	$\uparrow$	lower bound better than no information	multiple eqs
10	privately	no	privately	no	$\uparrow$	$p_1 = 1, p_2 = 1,$ $q_1 = 1, q_2 = 0.5$	unique symmetric eq

Table 1: Best possible revenue impacts and corresponding signal structures in the interdependent value setting. c_i indicates signaling the common attribute to Bidder i and pr_i indicates signaling the private attribute to Bidder i. For the common attribute, "publicly" means the realization of the signal can be observed by all bidders and "privately" means the realization of the signal can only be observed by the corresponding bidder. Since private values are independent, whether that signal is revealed publicly or privately makes no difference. Note that the order of the two bidders is arbitrary, but the existence of the asymmetry is not.

$t_h) = \frac{1}{2}$. If the bidder is type t_h (high tech firm), then her valuation is only dependent on her private attribute, that is $w_i = G$ with value $\alpha > 1$ (pure strategy Nash equilibrium is not guaranteed if $\alpha = 1$) and $w_i = B$ with value 0. If the bidder is type t_l (low tech firm), her valuation is dependent on both common and private attributes: the bidder's value is 0 if both the common and her private attribute are B, and 1 if both are G. Formally,

$$i \in \{1, 2\}$$
$$v_i(w_0, w_i, t_i = t_l) = \begin{cases} 1, & \text{if } w_0 = G, w_i = G, \\ 0, & \text{else,} \end{cases}$$
$$v_i(w_0, w_i, t_i = t_h) = \begin{cases} \alpha, & \text{if } w_i = G, \\ 0, & \text{else,} \end{cases}$$

where $P(t_i = t_l) = \frac{1}{2}$ and $\alpha > 1$.

Analysis Before the game, the seller needs to decide which attribute she wants to signal to each bidder and whether the realization of the signal is public or private. The seller still provides one signal to each bidder, but the realization of that signal can be public. The complete results characterizing the best possible revenue impact and the corresponding signal structure based on seller's strategy is shown in Table 1. The main results to note are that there are two signal structures that are revenue enhancing. For brevity, we defer the relatively simple proofs of the negative results in cases one through eight to a longer version of this paper, and focus on the two positive outcomes.

When we allow one bidder (w.lo.g. Bidder 2) to observe a signal of her private attribute while the other bidder receives a private signal of the common attribute (case 9), there exists a revenue-enhancing signal structure. In equilibrium, a bidder of type t_h always bids her expected value given the signal realization of private attribute if she receives one. If Bidder 1 is type t_l she bids her expected value given the signal realization she observes. If Bidder 2 is type t_l, if she observes a *low* signal, her bid falls in the range $[\mathbb{E}[v|s_1^0 = L, s_2^2 = L], \mathbb{E}[v|s_1^0 = H, s_2^2 = L]]$ under Assumption 2 and also needs to be smaller than Bidder 1's expected value given Bidder 1 observes a *low* signal $\mathbb{E}[v|s_1^0 = L]$; if she observes a *high* signal, from Assumption 2 her bid falls in the range $[\mathbb{E}[v|s_1^0 = L, s_2^2 = H], \mathbb{E}[v|s_1^0 = H, s_2^2 = H]]$, and also needs to be greater than bidder 1's expected value given Bidder 1 observes a *high* signal $\mathbb{E}[v|s_1^0 = H]$.

Now, suppose the seller chooses signal structure $p_1 \in [0.5, 1], p_2 = 1, q_1 = 1, q_2 = 0.5$. If Bidder 1 observes a high signal, she knows with certainty that the common attribute is good, and is uncertain otherwise. Bidder 2 knows that her private attribute is bad if she observes a low signal, and is uncertain otherwise. Combined with the observation about bid ranges above, it now becomes a simple matter of algebra to show that the expected revenue is greater than that which is achieved when the seller reveals no information or full information, yielding the following theorem:

Theorem 3 *Privately revealing the realization of the common attribute signal to one bidder and privately revealing the realization of the private attribute signal to the other bidder, the seller's expected revenue at* $p_1 \in [0.5, 1], p_2 = 1, q_1 = 1, q_2 = 0.5$ *is always better than that she can achieve when revealing no information or full information.*

Proof: see Appendix [1] Proof of Theorem 3. An interesting observation about this signal structure is that, while the signal structure conveys more information to Bidder 1, her

[1] See online appendix.

Types (t_1, t_2)	Exp. revenue	Exp. revenue without signaling
t_h, t_h	0.8α	0.8α
t_h, t_l	0.72	0.64
t_l, t_h	0.64	0.64
t_l, t_l	0.64	0.64

Table 2: Revenue comparison between the cases with and without signaling when the auctioneer reveals the realization of common attribute privately to each bidder.

utility is actually lower compared with when there is no information. Bidder 2's utility improves.

Finally, we see what happens if the seller signals the common attribute to each bidder privately (case 10 in Table 1). In this situation, the equilibrium bidding strategy for t_h type bidder is to bid her expected value regardless of the signal she receives and for t_l type bidder is to bid her expected value conditioned on the other bidder observing same signal. It is easy to show that the signal structure $p_1 = p_2 = 1, q_1 = 1, q_2 = 0.5$, results in higher expected revenue than when the seller conveys no information or full information.

Theorem 4 *When revealing the signal realization of the common attribute privately to each bidder, the seller's revenue is higher at signal structures $p_1 = p_2 = 1, q_1 = 1, q_2 = 0.5$, or $p_1 = p_2 = 1, q_1 = 0.5, q_2 = 1$, than when revealing no information or full information.*

Proof: see Appendix Proof of Theorem 4. Consider signal structure $p_1 = p_2 = 1, q_1 = 1, q_2 = 0.5$ (the other one is symmetric). Bidder 1 always has perfect information. If Bidder 2 receives a low signal, she is certain w_0 is bad; however, she is uncertain when she gets a high signal. Surprisingly, although Bidder 1 has perfect information, her expected utility is actually lower than that of Bidder 2. It is easy to see that if both bidders are t_l types or t_h types, then the expected utility of each bidder is zero. The interesting case is when Bidder 1 is t_h type, and Bidder 2 is t_l type. In this situation, Bidder 2, who has imperfect information is more likely to receive a high signal than Bidder 1; therefore, in expectation, the perfect information bidder will pay more (since it is a second price auction), hurting her utility. The utility of each bidder under this signal structure can be calculated as

$$\begin{aligned}
u_1 =& 0.25(P(s_2^0 = L)(x\alpha - \mathbb{E}[v|s_1^0 = L, s_2^0 = L]) \\
&+ P(s_2^0 = H)(x\alpha - \mathbb{E}[v|s_1^0 = H, s_2^0 = H])), \\
u_2 =& 0.25(P(s_1^0 = L)(x\alpha - \mathbb{E}[v|s_1^0 = L, s_2^0 = L]) \\
&+ P(s_1^0 = H)(x\alpha - \mathbb{E}[v|s_1^0 = H, s_2^0 = H])),
\end{aligned}$$

where $P(s_2^0 = L) < P(s_1^0 = L)$ and $P(s_2^0 = H) > P(s_1^0 = H)$.

The following example demonstrates the above analysis. Let $P(w_0 = G) = P(w_1 = G) = P(w_2 = G) = 0.8$. Table 2 shows the revenue comparison between the cases with and without signaling. As $p_1 = p_2 = 1, q_1 = 1, q_2 = 0.5$, we get

$$P(s_2^0 = L) = 0.1 < P(s_1^0 = L) = 0.2$$

and

$$P(s_2^0 = H) = 0.9 > P(s_1^0 = H) = 0.8.$$

Thus, the revenue increase is from when Bidder 1 is a t_h type, while Bidder 2 is a t_l type and receives a high signal (row 2 in Table 2). Bidder 2's receipt of a high signal hurts Bidder 1, as she is the one who has to pay more!

Conclusion

The key point in the emerging signaling literature in information economics and computer science is to study what can be achieved through information design, or persuasion, when the mechanism is already fixed. We demonstrate the range of possible outcomes that can be achieved through different signaling schemes in common value auction, and show that the uninformative scheme has the lowest risk among those that extract full surplus. While different signal structures may not help improve revenue in second-price sealed bid common value auctions, there are natural auction models, like the interdependent value model for corporate takeovers we present, in which the optimal design of signal structures can be revenue enhancing.

References

[Abraham et al. 2011] Abraham, I.; Athey, S.; Babaioff, M.; and Grubb, M. 2011. Peaches, lemons, and cookies: Designing auction markets with dispersed information. Technical report.

[Alkoby, Sarne, and Milchtaich 2017] Alkoby, S.; Sarne, D.; and Milchtaich, I. 2017. Strategic signaling and free information disclosure in auctions. In *Proc. AAAI*.

[Bergemann and Pesendorfer 2007] Bergemann, D., and Pesendorfer, M. 2007. Information structures in optimal auctions. *Journal of Economic Theory* 137(1):580 – 609.

[Bergemann, Brooks, and Morris 2017] Bergemann, D.; Brooks, B.; and Morris, S. 2017. First-price auctions with general information structures: Implications for bidding and revenue. *Econometrica* 85(1):107–143.

[Berkovitch and Khanna 1991] Berkovitch, E., and Khanna, N. 1991. A theory of acquisition markets: Mergers versus tender offers, and golden parachutes. *Review of Financial Studies* 4(1):149–174.

[Brinkman, Wellman, and Page 2014] Brinkman, E.; Wellman, M. P.; and Page, S. E. 2014. Signal structure and strategic information acquisition: Deliberative auctions with interdependent values. In *Proc. AAMAS*, 229–236.

[Bro Miltersen and Sheffet 2012] Bro Miltersen, P., and Sheffet, O. 2012. Send mixed signals: Earn more, work less. In *Proc. ACM EC*, 234–247.

[Celis et al. 2012] Celis, L. E.; Karlin, A. R.; Leyton-Brown, K.; Nguyen, C. T.; and Thompson, D. R. M. 2012. Approximately revenue-maximizing auctions for deliberative agents. In *Proc. AAAI*, 1313–1318.

[Chhabra, Das, and Sarne 2014] Chhabra, M.; Das, S.; and Sarne, D. 2014. Competitive information provision in sequential search markets. In *Proc. AAMAS*, 565–572.

[Das and Kamenica 2015] Das, S., and Kamenica, E. 2015. Representations of information structures. In *Allerton Conf. on Communication, Control, and Computing*, 737–743.

[Das and Li 2014] Das, S., and Li, Z. 2014. The role of common and private signals in two-sided matching with interviews. In *Proc. WINE*, 492–497.

[Das, Kamenica, and Mirka 2017] Das, S.; Kamenica, E.; and Mirka, R. 2017. Reducing congestion through information design. In *Proceedings of the 55th Allerton Conference on Communication, Control, and Computing*, 1279–1284.

[Daskalakis, Papadimitriou, and Tzamos 2016] Daskalakis, C.; Papadimitriou, C.; and Tzamos, C. 2016. Does information revelation improve revenue? In *Proc. ACM EC*, 233–250.

[Dughmi 2018] Dughmi, S. 2018. On the hardness of designing public signals. *Games and Economic Behavior*.

[Emek et al. 2012] Emek, Y.; Feldman, M.; Gamzu, I.; Paes Leme, R.; and Tennenholtz, M. 2012. Signaling schemes for revenue maximization. In *Proc. ACM EC*, 514–531.

[Gentzkow and Kamenica 2014] Gentzkow, M., and Kamenica, E. 2014. Costly persuasion. *American Economic Review* 104(5):457–62.

[Gentzkow and Kamenica 2016] Gentzkow, M., and Kamenica, E. 2016. Bayesian persuasion with multiple senders and rich signal spaces. *Working paper*.

[Gilson and Schwartz 2005] Gilson, R. J., and Schwartz, A. 2005. Understanding macs: Moral hazard in acquisitions. *The Journal of Law, Economics, and Organization* 21(2):330–358.

[Goeree and Offerman 2003] Goeree, J. K., and Offerman, T. 2003. Competitive bidding in auctions with private and common values. *The Economic Journal* 113(489):598–613.

[Guo and Deligkas 2013] Guo, M., and Deligkas, A. 2013. Revenue maximization via hiding item attributes. In *Proc. IJCAI*, 157–163.

[Hajaj and Sarne 2014] Hajaj, C., and Sarne, D. 2014. Strategic information platforms: Selective disclosure and the price of "free". In *Proc. ACM EC*, 839–856.

[Hausch 1987] Hausch, D. B. 1987. An asymmetric common-value auction model. *The RAND Journal of Economics* 18(4):611–621.

[Kamenica and Gentzkow 2011] Kamenica, E., and Gentzkow, M. 2011. Bayesian persuasion. *The American Economic Review* 101(6):2590–2615.

[Knecht and Calenbuhr 2007] Knecht, F., and Calenbuhr, V. 2007. Using capital transaction due diligence to demonstrate csr assessment in practice. *Corporate Governance: The international journal of business in society* 7(4):423–433.

[Krishna 2002] Krishna, V. 2002. *Auction Theory*. Academic Press.

[Larson and Sandholm 2001a] Larson, K., and Sandholm, T. 2001a. Bargaining with limited computation: Deliberation equilibrium. *Artificial Intelligence* 132(2):183 – 217.

[Larson and Sandholm 2001b] Larson, K., and Sandholm, T. 2001b. Costly valuation computation in auctions. In *Proc. TARK*, 169–182.

[Larson and Sandholm 2005] Larson, K., and Sandholm, T. 2005. Mechanism design and deliberative agents. In *Proc. AAMAS*, 650–656.

[Milgrom and Weber 1982a] Milgrom, P. R., and Weber, R. J. 1982a. A theory of auctions and competitive bidding. *Econometrica* 1089–1122.

[Milgrom and Weber 1982b] Milgrom, P. R., and Weber, R. J. 1982b. The value of information in a sealed-bid auction. *Journal of Mathematical Economics* 10(1):105–114.

[Ostrovsky and Schwarz 2010] Ostrovsky, M., and Schwarz, M. 2010. Information Disclosure and Unraveling in Matching Markets. *American Economic Journal: Microeconomics* 2(2):34–63.

[Parreiras 2006] Parreiras, S. O. 2006. Affiliated common value auctions with differential information: the two bidder case. *Contributions in Theoretical Economics* 6(1):1–19.

[Rabinovich et al. 2015] Rabinovich, Z.; Jiang, A. X.; Jain, M.; and Xu, H. 2015. Information disclosure as a means to security. In *Proc. AAMAS*, 645–653.

[Rayo and Segal 2010] Rayo, L., and Segal, I. 2010. Optimal information disclosure. *Journal of Political Economy* 118(5):949–987.

[Reuer, Tong, and Wu 2012] Reuer, J. J.; Tong, T. W.; and Wu, C.-W. 2012. A signaling theory of acquisition premiums: Evidence from ipo targets. *Academy of Management Journal* 55(3):667–683.

[Shen, Tang, and Zeng 2018] Shen, W.; Tang, P.; and Zeng, Y. 2018. A closed-form characterization of buyer signaling schemes in monopoly pricing. In *AAMAS*.

[Skreta 2011] Skreta, V. 2011. On the informed seller problem: optimal information disclosure. *Review of Economic Design* 15(1):1–36.

[Syrgkanis, Kempe, and Tardos 2015] Syrgkanis, V.; Kempe, D.; and Tardos, E. 2015. Information asymmetries in common-value auctions with discrete signals. In *Proc. ACM EC*, 303–303.

[Thompson and Leyton-Brown 2007] Thompson, D. R. M., and Leyton-Brown, K. 2007. Valuation uncertainty and imperfect introspection in second-price auctions. In *Proc. AAAI*, 148–153.

[Thompson and Leyton-Brown 2011] Thompson, D. R. M., and Leyton-Brown, K. 2011. Dominant-strategy auction design for agents with uncertain, private values. In *Proc. AAAI*, 745–750.

[Wilson 1967] Wilson, R. B. 1967. Competitive bidding with asymmetric information. *Management Science* 13(11):816–820.

[Xu et al. 2016] Xu, H.; Freeman, R.; Conitzer, V.; Dughmi, S.; and Tambe, M. 2016. Signaling in bayesian stackelberg games. In *AAMAS*.

Proof of Theorem 3

Theorem 3 *Privately revealing the realization of the common attribute signal to one bidder and privately revealing the realization of the private attribute signal to the other bidder, the seller's expected revenue at* $p_1 \in [0.5, 1], p_2 = 1, q_1 = 1, q_2 = 0.5$ *is always better than that she can achieve when revealing no information or full information.*

Proof Without loss of generality, we assume Bidder 1 receives a private signal of the common attribute while Bidder 2 observes a signal of her private attribute.

If Bidder 1 is type t_h, she bids $\text{bid}^1_{t_h} = \alpha x$ since she only receives a signal of the common value. If Bidder 1 is type t_l, sice the private attribute of Bidder 2 does not influence the value of Bidder 1, if she observes a *low* signal, she bids $\text{bid}^1_{t_l}(L) = \mathbb{E}[v|s^0_1 = L]$; if she observes a *high* signal, she bids $\text{bid}^1_{t_h}(L) = \mathbb{E}[v|s^0_1 = H]$.

If Bidder 2 is type t_h, she bids her expected value given the signal realization of the private attribute $\text{bid}^2_{t_h} = \mathbb{E}(v|s^2_2)$. If Bidder 2 is type t_l, if she observes a *low* signal, her equilibrium bid falls in the range

$$\text{bid}^2_{t_l}(L) \in [\mathbb{E}[v|s^0_1 = L, s^2_2 = L], \mathbb{E}[v|s^0_1 = H, s^2_2 = L]],$$
$$\text{and } \text{bid}^2_{t_l}(L) \leq \mathbb{E}[v|s^0_1 = L]$$

under Assumption 2; if she observes a *high* signal, from Assumption 2, her equilibrium bid falls in the range

$$\text{bid}^2_{t_l}(H) \in [\mathbb{E}[v|s^0_1 = L, s^2_2 = H], \mathbb{E}[v|s^0_1 = H, s^2_2 = H]],$$
$$\text{and } \text{bid}^2_{t_l}(H) \geq \mathbb{E}[v|s^0_1 = H].$$

Considering the lower bound of the revenue, we choose the minimum bid under all cases. We separate the revenue into the following four parts,

1. $t_1 = t_l$ and $t_2 = t_l$, where we have $P(t_1 = t_l, t_2 = t_l) = \frac{1}{4}$:

$$\begin{aligned} R_1 =& P(s^0_1 = H)P(s^2_2 = H)\mathbb{E}[v|s^0_1 = H] \\ &+ P(s^0_1 = H)P(s^2_2 = L)\mathbb{E}[v|s^0_1 = L, s^2_2 = L] \\ &+ P(s^0_1 = L)P(s^2_2 = H)\mathbb{E}[v|s^0_1 = L] \\ &+ P(s^0_1 = L)P(s^2_2 = L)\mathbb{E}[v|s^0_1 = L, s^2_2 = L]; \end{aligned}$$

2. $t_1 = t_l$ and $t_2 = t_h$, where we have $P(t_1 = t_l, t_2 = t_h) = \frac{1}{4}$:

$$\begin{aligned} R_2 =& P(s^0_1 = H)P(s^2_2 = H) \\ & \min(\mathbb{E}[v|s^0_1 = H], \mathbb{E}(v|s^2_2 = H)) \\ &+ P(s^0_1 = H)P(s^2_2 = L) \\ & \min(\mathbb{E}[v|s^0_1 = H], \mathbb{E}(v|s^2_2 = L)) \\ &+ P(s^0_1 = L)P(s^2_2 = H) \\ & \min(\mathbb{E}[v|s^0_1 = L], \mathbb{E}(v|s^2_2 = H)) \\ &+ P(s^0_1 = L)P(s^2_2 = L) \\ & \min(\mathbb{E}[v|s^0_1 = L], \mathbb{E}(v|s^2_2 = L)); \end{aligned}$$

3. $t_1 = t_h$ and $t_2 = t_l$, where we have $P(t_1 = t_h, t_2 = t_l) = \frac{1}{4}$:

$$\begin{aligned} R_3 =& P(s^2_2 = H)\max(\mathbb{E}[v|s^0_1 = L, s^2_2 = H], \mathbb{E}[v|s^0_1 = H]) \\ &+ P(s^2_2 = L)\mathbb{E}[v|s^0_1 = L, s^2_2 = L]; \end{aligned}$$

4. $t_1 = t_h$ and $t_2 = t_h$, $P(t_1 = t_h, t_2 = t_h) = \frac{1}{4}$:

$$\begin{aligned} R_4 =& P(s^2_2 = H)\min(\alpha x, \mathbb{E}(v|s^2_2 = H)) \\ &+ P(s^2_2 = L)\min(\alpha x, \mathbb{E}(v|s^2_2 = L)). \end{aligned}$$

The expected revenue is $R = 0.25(R_1 + R_2 + R_3 + R_4)$. By simple algebra, we find that the proposed signal structure achieves better revenue than either no information or full information.

Proof of Theorem 4

Theorem 4 *When revealing the signal realization of the common attribute privately to each bidder, the seller's revenue is higher at signal structures* $p_1 = p_2 = 1, q_1 = 1, q_2 = 0.5$*, or* $p_1 = p_2 = 1, q_1 = 0.5, q_2 = 1$*, than when revealing no information or full information.*

Proof Since the private attribute of each bidder does not affect the value to the other bidder, if $t_i = t_l$, the equilibrium strategy of bidder i is the same as in Proposition 1.

$$\begin{aligned} \text{bid}^i_L(L) =& \mathbb{E}(v|s^0_i = L, s^0_{-i} = L) \\ =& v(w_0 = G, w_i = G) \\ & P(w_0 = G|s^0_i = L, s^0_{-i} = L)P(w_i = G) \\ &+ v(w_0 = G, w_i = B) \\ & P(w_0 = G|s^0_i = L, s^0_{-i} = L)P(w_i = B) \\ &+ v(w_0 = B, w_i = G) \\ & P(w_0 = B|s^0_i = L, s^0_{-i} = L)P(w_i = G) \\ &+ v(w_0 = B, w_i = B) \\ & P(w_0 = B|s^0_i = L, s^0_{-i} = L)P(w_i = B) \\ =& \frac{(1-p_1)(1-p_2)x^2}{(1-p_1)(1-p_2)x + q_1 q_2(1-x)}, \\ \text{bid}^i_H(H) =& \mathbb{E}(v|s^0_i = H, s^0_{-i} = H) \\ =& \frac{p_1 p_2 x^2}{p_1 p_2 x + (1-q_1)(1-q_2)(1-x)}. \end{aligned}$$

If $t_i = t_h$, the bidder's payoff is only related to the relative private attribute, so the equilibrium bidding strategy is always

$$\text{bid}^i_{t_h} = \alpha x.$$

We break the revenue up into the following four parts,

1. $t_1 = t_l$ and $t_2 = t_l$, where we have $P(t_1 = t_l, t_2 = t_l) = \frac{1}{4}$:

$$\begin{aligned} R_1 =& P(s^0_1 = H, s^0_2 = H)\text{bid}^i_H(H) \\ &+ (1 - P(s^0_1 = H, s^0_2 = H))\text{bid}^i_L(L); \end{aligned}$$

2. $t_1 = t_l$ and $t_2 = t_h$, where we have $P(t_1 = t_l, t_2 = t_h) = \frac{1}{4}$:

$$\text{As } \alpha > 1, \alpha x > \text{bid}_H(H);$$
$$R_2 = P(s_1^0 = H)\text{bid}_H^1(H) + P(s_1^0 = L)\text{bid}_L^1(L);$$

3. $t_1 = t_h$ and $t_2 = t_l$, where we have $P(t_1 = t_h, t_2 = t_l) = \frac{1}{4}$:

$$R_3 = P(s_2^0 = H)\text{bid}_H^1(H) + P(s_2^0 = L)\text{bid}_L^2(L);$$

4. $t_1 = t_h$ and $t_2 = t_h$, $P(t_1 = t_h, t_2 = t_h) = \frac{1}{4}$:

$$R_4 = \alpha x.$$

Thus, the expected revenue is $R = \frac{1}{4}(R_1 + R_2 + R_3 + R_4)$. By simple algebra, we can find that the signal structure $p_1 = p_2 = 1, q_1 = 1, q_2 = 0.5$ yields higher revenue than no information or full information, and the revenue increase is from R_3.